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Engineering Notes

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Acceleration-Compensated Zero-Effort-Miss Guidance Law

Vincent C. Lam*
Lockheed Martin Missiles and Fire Control, Grand Prairie, Texas 75051

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Introduction

PROPORTIONAL navigation guidance has been widely used for many of the missile guidance laws. There are several variations of proportional navigation guidance [1,2]. When the acceleration command is applied perpendicular to the missile-to-target line of sight (LOS), it is called true proportional navigation guidance (TPNG). When the acceleration command is applied perpendicular to the pursuer velocity vector, it is called pure proportional navigation guidance (PPNG). Augmented proportional navigation guidance [3] (APNG) is basically defined as proportional navigation guidance plus a target acceleration term to compensate for the target acceleration.

Although it has been discussed in [4,5] that proportional navigation is mathematically equivalent to zero-effort-miss guidance (ZEMG), only a planar motion problem was illustrated. We will show how TPNG can be derived directly from ZEMG. We will also show why APNG does not fully take advantage of the target acceleration information. An acceleration-compensated zero-effort-miss guidance law (ACZEMG) has been developed that takes full advantage of the knowledge of target acceleration. Two examples are included to compare the performances of TPNG, APNG, and ACZEMG for a constantly accelerating target.

Zero-Effort-Miss Guidance Law and True Proportional Navigation Guidance

In this section, we will prove that zero-effort-miss guidance is the same as true proportional navigation guidance. The proportional guidance acceleration command is proportional to the range rate and LOS angular velocity. Let $\mathbf{r}$ be the current missile-to-target (target relative to missile) position vector and r be the current missile-to-target range. The current missile-to-target position vector can be written as

$$\mathbf{r} = r\mathbf{e}_1 \tag{1}$$

where $\mathbf{e}_1$ is the LOS unit vector. The current missile-to-target velocity $\mathbf{v}$ is

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_1 + \boldsymbol{\omega} \times \mathbf{r} \tag{2}$$

where $\dot{r}$ is the range rate and $\boldsymbol{\omega}$ is the LOS angular velocity.

For TPNG, the acceleration command is applied perpendicular to the LOS direction. In this case, the TPNG command [6,7] can be expressed as

$$\mathbf{a}_c = K_N \dot{r}\mathbf{e}_1 \times \boldsymbol{\omega} \tag{3}$$

where $\mathbf{a}_c$ is the acceleration command and K_N is the proportional navigation constant. It is obvious that

$$\mathbf{a}_c \cdot \mathbf{e}_1 = K_N \dot{r}(\mathbf{e}_1 \times \boldsymbol{\omega}) \cdot \mathbf{e}_1 = 0 \tag{4}$$

So the acceleration command is perpendicular to the LOS vector.

For the time being, assume neither the missile nor the target is accelerating. The missile-to-target position vector at time t is

$$\mathbf{x} = \mathbf{r} + \mathbf{v}t \tag{5}$$

Let τ be the time to go, that is, the time that the distance between the missile and target is at a minimum and the corresponding ZEM position vector is

$$\mathbf{z} = \mathbf{r} + \mathbf{v}\tau \tag{6}$$

The ZEMG is simply

$$\mathbf{a}_c = K_Z \frac{\mathbf{z}}{\tau^2} = K_Z \frac{\mathbf{r} + \mathbf{v}\tau}{\tau^2} \tag{7}$$

where K_Z is the guidance constant. We will use the following time-to-go definition [8]:

$$\tau = -\frac{r}{\dot{r}} \tag{8}$$

By substituting Eq. (8) into Eq. (7), we have

$$\mathbf{a}_c = K_Z \left\{ \frac{\mathbf{r}}{\tau^2} + \frac{\mathbf{v}}{\tau} \right\} = K_Z \left\{ \frac{\dot{r}^2 \mathbf{r}}{r^2} - \frac{\dot{r}\mathbf{v}}{r} \right\} \tag{9}$$

With the help of Eq. (2), we can express Eq. (9) as

$$\mathbf{a}_c = K_Z \left\{ \frac{\dot{r}^2 \mathbf{r}}{r^2} - \frac{\dot{r}(\dot{r}\mathbf{e}_1 + \boldsymbol{\omega} \times \mathbf{r})}{r} \right\} = -K_Z \frac{\dot{r}\boldsymbol{\omega} \times \mathbf{r}}{r} = K_Z \frac{\dot{r}\mathbf{r} \times \boldsymbol{\omega}}{r} = K_Z \dot{r}\mathbf{e}_1 \times \boldsymbol{\omega} \tag{10}$$

As we can see, by using the time to go defined in Eq. (8) the acceleration command of the ZEMG is automatically perpendicular to the LOS. Furthermore, the ZEMG command in Eq. (10) is identical to the TPNG command defined in Eq. (3). The ZEMG in this case is not equivalent to other forms of proportional navigation guidance, such as PPNG.

Another time to go from [8] is defined as

$$\tau = -\frac{\mathbf{v} \cdot \mathbf{r}}{\mathbf{v} \cdot \mathbf{v}} \tag{11}$$

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*Currently Senior Principal Systems Engineer, Raytheon Missile Systems, Tucson, AZ.

The corresponding ZEM vector [8] is

$$\mathbf{z} = \frac{(\mathbf{v}\cdot\mathbf{v})\mathbf{r} - (\mathbf{v}\cdot\mathbf{r})\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}} = \frac{(\mathbf{v}\times\mathbf{r})\times\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}} \tag{12}$$

Using the time-to-go definition from Eq. (11), the ZEMG law in this case is

$$\mathbf{a}_c = K_Z\frac{\mathbf{z}}{\tau^2} = \frac{K_Z(\mathbf{v}\cdot\mathbf{v})^2}{(\mathbf{v}\cdot\mathbf{r})^2}\frac{(\mathbf{v}\cdot\mathbf{v})\mathbf{r} - (\mathbf{v}\cdot\mathbf{r})\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}} = K_Z\frac{v^2}{(\mathbf{v}\cdot\mathbf{r})^2}(\mathbf{v}\times\mathbf{r})\times\mathbf{v} \tag{13}$$

Let

$$\mathbf{v}\cdot\mathbf{r} = vr\cos\alpha \tag{14}$$

where α is the angle between $\mathbf{v}$ and $\mathbf{r}$. Using Eq. (14), we can rewrite Eq. (13) as

$$\mathbf{a}_c = K_Z\left\{\frac{v^2\mathbf{r}}{r^2\cos^2\alpha} + \frac{v\mathbf{v}}{r\cos\alpha}\right\} = K_Z\frac{(\mathbf{v}\times\mathbf{r})\times\mathbf{v}}{r^2\cos^2\alpha} \quad \text{for } \cos\alpha \neq 0 \tag{15}$$

If we choose the acceleration perpendicular to the LOS, then the ZEMG acceleration command is reduced to

$$\mathbf{a}_c = \frac{K_Z v(\mathbf{e}_1\times\boldsymbol{\omega})}{\cos\alpha} = \frac{K_Z\dot{r}(\mathbf{e}_1\times\boldsymbol{\omega})}{\cos^2\alpha} \quad \text{for } \cos\alpha \neq 0 \tag{16}$$

Acceleration-Compensated Zero-Effort-Miss Guidance and Augmented Proportional Navigation Guidance

If we modify the ZEMG from Eq. (6) to include the relative acceleration, the ZEM miss distance position vector is

$$\mathbf{z} = \mathbf{r} + \mathbf{v}\tau + \tfrac{1}{2}\mathbf{a}\tau^2 \tag{17}$$

where $\mathbf{a}$ is the missile-to-target acceleration. In this case, the ZEMG acceleration command is

$$\mathbf{a}_c = K_Z\frac{\mathbf{z}}{\tau^2} = K_Z\frac{\mathbf{r} + \mathbf{v}\tau + \frac{1}{2}\mathbf{a}\tau^2}{\tau^2} = K_Z\left\{\frac{\mathbf{r} + \mathbf{v}\tau}{\tau^2} + \frac{1}{2}\mathbf{a}\right\} \tag{18}$$

If we use the time-to-go definition from Eq. (8), then Eq. (18) becomes

$$\mathbf{a}_c = K_Z\{\dot{r}\mathbf{e}_1\times\boldsymbol{\omega} + \tfrac{1}{2}\mathbf{a}\} \tag{19}$$

The first term on the right-hand side of Eq. (19) is the TPNG acceleration command and the second term compensates for the target acceleration. Equation (19) is known as augmented proportional navigation guidance. In essence, the APNG is based on the ZEM position vector that accounts for the acceleration, but the time to go does not take into account the acceleration.

To take full advantage of the knowledge of acceleration, the time to go should be computed using the acceleration information. The time to go that accounts for the acceleration can be obtained by solving the following equation [8]:

$$\tfrac{1}{2}\mathbf{a}\cdot\mathbf{a}\tau^3 + \tfrac{3}{2}\mathbf{a}\cdot\mathbf{v}\tau^2 + (\mathbf{a}\cdot\mathbf{r} + \mathbf{v}\cdot\mathbf{v})\tau + \mathbf{v}\cdot\mathbf{r} = 0 \tag{20}$$

Once the time to go is solved, the ACZEMG is simply

$$\mathbf{a}_c = K_Z\left\{\frac{\mathbf{r}}{\tau^2} + \frac{\mathbf{v}}{\tau} + \frac{\mathbf{a}}{2}\right\} \tag{21}$$

To account for the acceleration uncertainty or maneuvering, we will multiply the acceleration by an acceleration confidence factor c in Eqs. (20) and (21). The results are

$$\frac{c^2}{2}\mathbf{a}\cdot\mathbf{a}\tau^3 + \frac{3c}{2}\mathbf{a}\cdot\mathbf{v}\tau^2 + (c\mathbf{a}\cdot\mathbf{r} + \mathbf{v}\cdot\mathbf{v})\tau + \mathbf{v}\cdot\mathbf{r} = 0 \tag{22}$$

$$\mathbf{a}_c = K_Z\left\{\frac{\mathbf{r}}{\tau^2} + \frac{\mathbf{v}}{\tau} + \frac{c\mathbf{a}}{2}\right\} \tag{23}$$

We can use the acceleration confidence factor to control the contribution of acceleration. If the acceleration data are very good, we can set $c = 1$. On the other hand, if the acceleration data are not reliable, we can set $c = 0$.

Another approach to compute the time to go is to project the relative velocity and acceleration along the LOS direction and solve for the time when the range is zero. Mathematically, we will solve the following equation:

$$\mathbf{z}\cdot\mathbf{r} = \tfrac{1}{2}\mathbf{a}\cdot\mathbf{r}\tau^2 + \mathbf{v}\cdot\mathbf{r}\tau + \mathbf{r}\cdot\mathbf{r} = \tfrac{1}{2}\ddot{r}\tau^2 + \dot{r}\tau + r = 0 \tag{24}$$

The solution is

$$\tau = \frac{2r}{-\dot{r} + \sqrt{\dot{r}^2 - 2r\ddot{r}}} \tag{25}$$

The time to go obtained from Eq. (20) is an exact solution of the predicted time for the missile to travel to the point of closest approach if both missile and target travel at constant acceleration. The time to go obtained from Eq. (25) is similar to the approach used in Eq. (2.7) from [8]. It represents the estimated time for the missile to travel to the line drawn from the target perpendicular to the current LOS vector.

If the confidence factor c is included, we have

$$\tau = \frac{2r}{-\dot{r} + \sqrt{\dot{r}^2 - 2c\,r\ddot{r}}} \tag{26}$$

If $c = 0$ and the range rate is negative, the time to go in Eq. (26) is the same as in Eq. (8), and in this case we have shown that the acceleration command using Eqs. (23) and (26) is the same as the TPNG acceleration command shown in Eq. (10).

Let the relative acceleration perpendicular to the LOS be

$$\bar{\mathbf{a}} = \mathbf{a} - \mathbf{a}\cdot\mathbf{e}_1\mathbf{e}_1 \tag{27}$$

It is clear that

$$\bar{\mathbf{a}}\cdot\mathbf{e}_1 = 0 \tag{28}$$

If the ACZEMG acceleration command is applied perpendicular to the LOS, the acceleration command from Eq. (23) is simplified as

$$\mathbf{a}_c = K_Z\left\{\frac{\boldsymbol{\omega}\times\mathbf{r}}{\tau} + \frac{c\bar{\mathbf{a}}}{2}\right\} \tag{29}$$

If Eq. (26) is used for the time to go, then Eq. (29) becomes

$$\mathbf{a}_c = \frac{K_Z}{2}\{(-\dot{r} + \sqrt{\dot{r}^2 - 2c\,r\ddot{r}})\boldsymbol{\omega}\times\mathbf{e}_1 + c\bar{\mathbf{a}}\} \tag{30}$$

If $c = 0$, then

$$\mathbf{a}_c = -K_Z\dot{r}\boldsymbol{\omega}\times\mathbf{e}_1 \tag{31}$$

Equation (31) is the same as the TPNG shown in Eq. (10).

Examples

In this section, we will compare the performances of ACZEMG, TPNG, and APNG. The following acceleration index will be used to compare the divert requirement:

$$a_I = \int_0^{t_f} \|\mathbf{a}_c\|\,\mathrm{d}t \tag{32}$$

Two examples assuming an ideal missile with perfect response are given here. In both examples, the missile is assumed to have a 10 g

Table 1 Missile and target initial conditions

	Missile	Target
Position, m	(0, 0, 10,000)	(10,000, 500, 12,000)
Velocity, m/s	(900, 0, 0)	(−200, 0, −100)

acceleration limit and the initial conditions of the simulations are summarized in Table 1. The first example assumes perfect target position, velocity, and acceleration data. The performances in terms of miss distance, acceleration magnitude, and acceleration index are compared against the TPNG, APNG, and ACZEMG performances. The second example is the same as the first example except that target acceleration noise is added to the APNG and ACZEMG simulations. The purpose of the second example is to show how well APNG and ACZEMG perform if the acceleration data are noisy.

Example 1

In this example, the target simply has a 5 g pull up. The missile and target motions are assumed to be perfectly known. The results of example 1 are shown in Figs. 1–3. Figure 1 indicates that the later portion of the trajectory of ACZEMG is closer to a straight line because the target acceleration is compensated for earlier. From Fig. 2, we can see that ACZEMG demands high acceleration at the beginning to compensate for the target acceleration, and as a result it demands less acceleration as the missile comes closer to the target. On the other hand, TPNG demands less acceleration at the beginning, but demands higher acceleration as the missile draws closer to the target. As expected, APNG is somewhat in between the other two because the target acceleration is partially compensated for by the

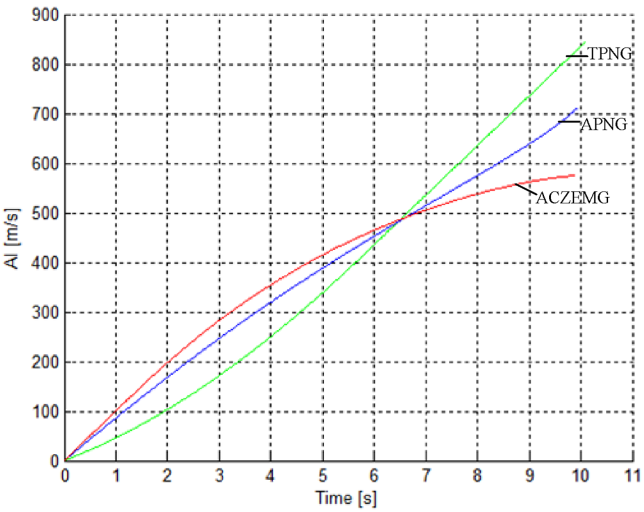

Fig. 3 Example 1 acceleration index.

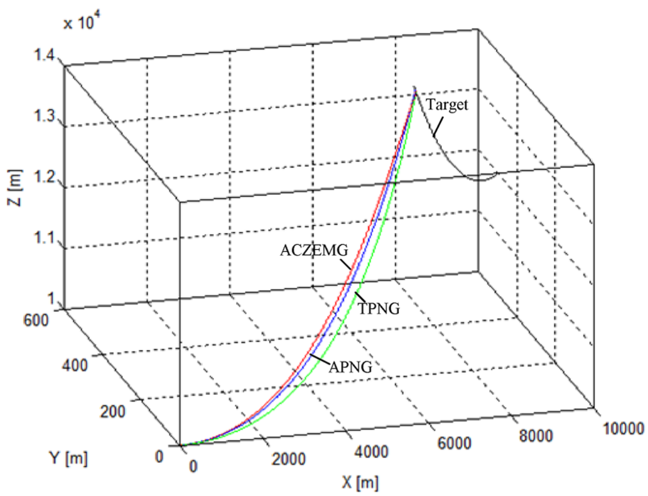

Fig. 1 Example 1 trajectory.

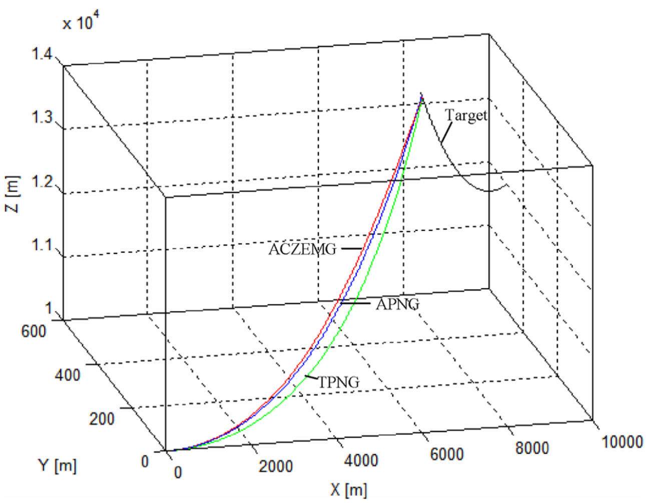

Fig. 4 Example 2 trajectory.

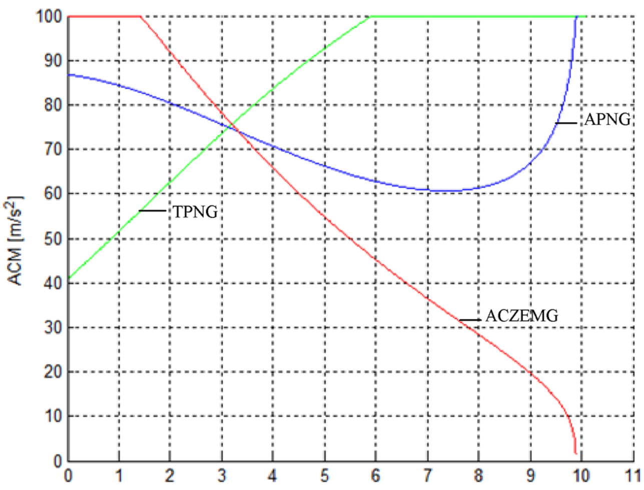

Fig. 2 Example 1 acceleration magnitude.

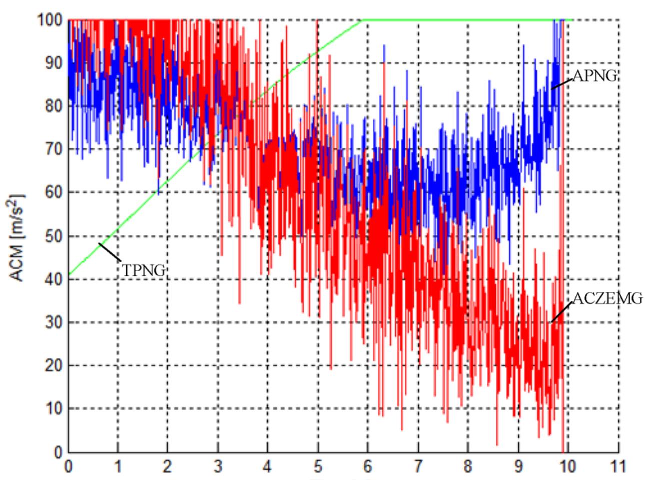

Fig. 5 Example 2 acceleration magnitude.

Table 2 Miss distance and acceleration index summary

	Miss distance, m			Acceleration index, m/s		
	TPNG	APNG	ACZEMG	TPNG	APNG	ACZEMG
Example 1	114.7313	0.0129	0.0001	846	713	574
Example 2	114.7313	0.0503	0.0026	846	717	606

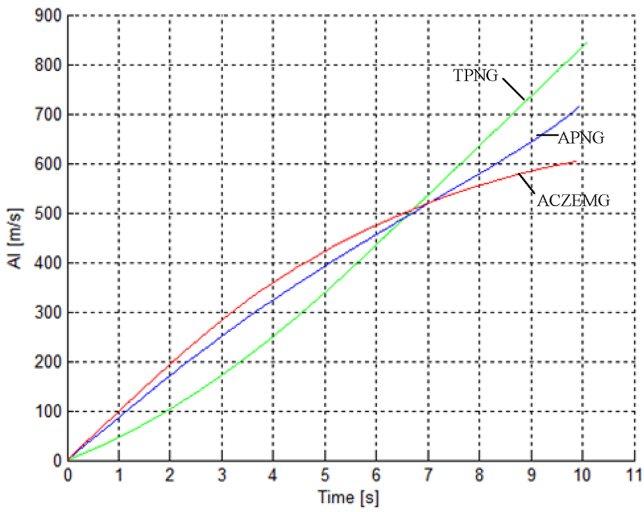

Fig. 6 Example 2 acceleration index.

acceleration term, as was discussed previously. Figure 3 shows that ACZEMG has the lowest acceleration index while TPNG has the highest acceleration index, and APNG is in between. The values of miss distance and acceleration index are shown in Table 2.

Example 2

This example is the same as example 1, except the target acceleration data are corrupted. Gaussian noise of 5 m/s^2 is added to the target acceleration for the APNG acceleration command while the target noise is added to the calculations of time to go and acceleration command for the ACZEMG. The trajectories are shown in Fig. 4 and are similar to those in Fig. 1. The acceleration command magnitudes for TPNG, APNG, and ACZEMG are shown in Fig. 5. Both APNG and ACZEMG acceleration command magnitudes are noisy, due to the acceleration noise. The position and velocity are assumed to be perfectly known, so the TPNG performance is not affected. The acceleration indexes are shown in Fig. 6. As shown in Table 2, the ACZEMG acceleration index is higher and so is the miss distance, but it is still better than TPNG and APNG.

Conclusions

We have derived TPNG directly from ZEMG, and we have shown why APNG does not take full advantage of the knowledge of acceleration. Against a target with constant acceleration, with perfect knowledge of that target acceleration, we have shown that ACZEMG outperforms TPNG and APNG in miss distance and acceleration index. The advantage of ACZEMG is that it uses the knowledge of the target acceleration and compensates for the target acceleration early on. One characteristic of ACZEMG is that it demands higher acceleration at the beginning and lower acceleration at the end. This is particularly advantageous for intercepting high altitude targets, where the missile divert capability is higher at low altitude and lower at high altitude. ACZEMG can make corrections to the collision path early so smaller divert is needed near interception.

There are situations when ACZEMG is less effective. If the acceleration data are noisy, the miss distance will increase and the acceleration index will increase. Particularly against maneuvering targets with noisy acceleration data, ACZEMG is expected to use more energy to compensate for the incorrect accelerations. At some point ACZEMG is not expected to outperform TPNG. To account for this, an acceleration confidence factor is multiplied against the acceleration term. This acceleration confidence factor approaches 1 when the acceleration estimate is excellent and it approaches 0 when the acceleration estimate is poor. When the acceleration confidence factor approaches 0, then the performance of ACZEMG is similar to the performance of TPNG.

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